DEFINING-EQUATION CONSTRUCTION IN HARDENING THEORY

B. P. Rusov

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The equation of state in hardening theory is most simply written [1, 2] as

$$\dot{p} = g(\sigma)/f(p), \tag{1}$$

where p is the creep strain and σ is the stress, while the dot denotes differentiation with respect to time.

The applicability of this equation has not finally been established, and there is no reasonably general method for determining the hardening function f(p). In practice, f(p) is usually determined by selection.

Here we use a special transformation of the creep curves in a method of determining the applicability of (1), and we propose a method of constructing f(p).

Experiment shows that the following equation applies over a not very wide range in stress and creep strain for metals [1, 2]:

$$\dot{p} = g(\sigma)/p^{\alpha}. \tag{2}$$

The application range of (2) is fairly narrow in some cases. It has been shown [3] that the hardening parameter α in (2) can be taken as constant for 30KhMA steel at 500°C only for small strains. At large creep strains, α is dependent on the stress, i.e., there is a deviation from (1). The same conclusion was reached in [4] for the creep in D-16T alloy.

Over 30 series of creep curves for various metals and alloys were examined in [5], which showed that in most cases only the initial parts of the curves can be represented as straight lines in logarithmic coordinates, and that for a stress range distinct for each material. Figure 1 shows creep curves for AMg5V alloy with ln p and ln t as axes recorded at 200°C [6]. The numbers on the curves are the stresses in MPa. The creep curves for this alloy do not become straight lines in logarithmic coordinates. Equation (2) cannot be applied to AMg5V even if one assumes that α is dependent on σ. It is necessary to define a new form for the equation, and it is not known whether one can use (1) to describe these experiments and others or whether it is necessary to formulate a more complicated model. It is necessary to devise methods of determining the applicability range of (1).

The method proposed here is based on describing the initial parts of the creep curves by means of (2) in a certain stress range.

We represent (1) in the form

$$\dot{p} = g(\sigma)/(p^{\alpha} - \varphi(p)), \tag{3}$$

where $\varphi(p) \ll p^{\alpha}$ at small p.

An individual creep curve can be described by (3) with any accuracy, by taking for example $\varphi(p)$ as a polynomial. The question is whether (3) enables one to describe a series of creep curves over sufficiently wide ranges in stress and strain.

We integrate (3) with $\sigma = \sigma_0 = \text{const}$

$$\int_0^p \varphi(p)\,dp = \frac{p^{\alpha+1}}{\alpha+1} - g(\sigma_0)\,t. \tag{4}$$

We put

$$\int_0^p \varphi(p)\,dp \equiv \Phi(p).$$

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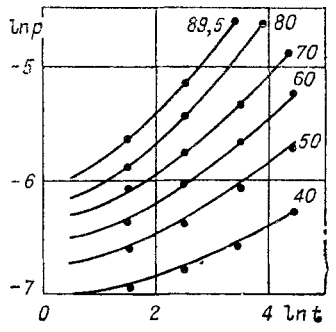

Fig. 1

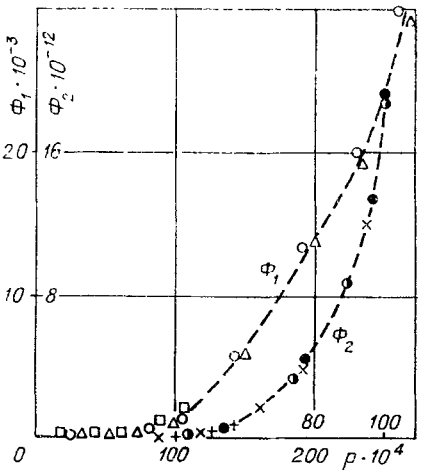

Fig. 2

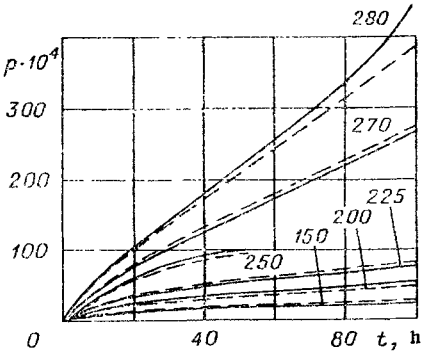

Fig. 3

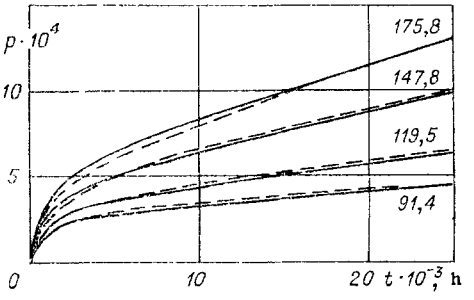

Fig. 4

If (3) applies (can describe the given series of creep curves), then the function

$$\Phi(p) = \frac{p^{\alpha+1}}{\alpha+1} - g(\sigma_0)\,t \tag{5}$$

should represent a single curve for different values of σ_0 and t. Here the stress σ_0 and time t appears as parameters. In the particular case where $\varphi(p) \equiv 0$, we obtain the power law of (2).

To construct the Φ—p relationship from (5), it is first necessary to find $g(\sigma)$, which can be determined by standard methods [1, 7, 8] by putting $\varphi(p) = 0$ in (4).

Figure 2 shows the function of (5) for steel 30KhMA (Φ_1) and AMg5V alloy (Φ_2). The experimental points correspond to individual creep curves. Figure 2 shows that the calculated values group around a single curve.

Similar constructions were performed for the above 30 series of creep curves [5]. In all cases, the values lie around a single curve in Φ—p axes, which indicates that (1) has a fairly wide range of application.

To determine f(p), it is first necessary to find $\varphi(p)$, i.e., by numerical differentiation of $\Phi(p)$. One can first approximate $\Phi(p)$ with a suitable function and then differentiate it to find $\varphi(p)$. When $f(p) = p^{\alpha} - \varphi(p)$ has been determined (the graph has been constructed), it can be approximated as a simpler function.

In many cases, good results are given by

$$f(p) = \mathrm{th}^{\alpha} ap. \tag{6}$$

For small p, the function of (6) degenerates into a power law, and (1) takes the form of (2). For $p \to \infty$, the function of (6) tends to one. Therefore, at sufficiently large creep strain, the strain rate $\dot{p}$ is determined only by the stress:

$$\dot{p} = g(\sigma). \tag{7}$$

Therefore, (1) with the hardening function of (6) describes the first and second creep stages.

If the creep curve has a pronounced second part, (7) enables one to derive $g(\sigma)$.

The curves for AMg5V alloy and 30KhMA steel showed that one can take $\alpha = 1$ for these materials.

In that case, the solution to (1) takes the form

$$p = \frac{1}{a} \operatorname{arch} e^{ag(\sigma_0)t}. \tag{8}$$

The points in Fig. 1 denote the values calculated from (8) ($a = 180$, $g(\sigma) = 3.64 \cdot 10^{-9} \sigma^5$); Fig. 3 (solid lines) shows the creep curves for 30KhMA steel [3], while the dot-dash lines show the calculated values ($a = 166.67$, $g(\sigma) = 2.408 \cdot 10^{-3} \exp(\sigma/29.4)$). It is evident from Figs. 1 and 3 that there is good agreement between the calculated and experimental values.

The solid lines in Fig. 4 show the creep curves for chromium-nickel-molybdenum steel at 450°C, where there is a pronounced second part [2]. The dot-dash lines show the calculated curves ($\alpha = 3$, $a = 2400$, $g(\sigma) = 8.14 \cdot 10^{-16} \sigma^{3.334}$). It is evident from Fig. 4 that in this case (1) with the hardening function of (6) gives a satisfactory description of the creep curves.

The above studies show that (1) has a fairly wide range. The agreement between the calculated values and experiment will be dependent on the accuracy in approximating the hardening function graph by the above method.

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